A Fuzzy-Wavelet Method for Analyzing Non-Stationary Time Series

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Abstract: *Fuzzy rule based systems are increasingly being used to deal with time series processes that may lack stochastic stability due to non-stationarity, multiscaling and persistent autocorrelations. Wavelet filtering can be used to deal with such phenomenon. A method for creating a fuzzy-rule base from a time series, where the first difference (returns) of the preprocessed series is used, and high frequency components have been removed, is reported. The performance of this system, trained using the fuzzy-wavelet method, is compared with a conventional fuzzy rule-based system trained on raw time series. The initial results appear encouraging in favour of the fuzzy-wavelet method.*

Keywords*:* *Artificial intelligence, fuzzy systems, wavelets, non-stationary, financial, time series, prediction*

1. Introduction

Time serial data of real-world phenomena is inherently non-stationary. Examples of non-stationary data can be found in financial trading instruments, including currencies, derivatives and shares [12], [20], and in astrophysical phenomena like sunspots [23]. The behaviour of non-stationary series is characterized by the observation that over a time period, one can neither find a clear tendency to return to a fixed value nor a linear trend. Time series usually have *seasonal variations*, *long-term* and *short-term fluctuations,* which may not be limited only to the mean of the series but may also affect its overall variance structure. Typically, such series are characterized by patterns like *trends* and localized abrupt changes, also known as *volatility* clustering. An important concept in time series analysis is "multiscaling" in which time series exhibit several phenomena, each occurring at different time horizons. Some time series are also characterized by a persistence of autocorrelations much longer than expected (long memory processes) and hence cannot be explained appropriately by ARIMA models. Since these series do not follow well-established theoretical phenomena, they are increasingly analyzed through the use of various 'model free' techniques.

2. Motivation

Artificial intelligence techniques such as genetic algorithms [16], neural networks [24] and hybrid statistical-neural approaches [19] have been reported for analysing both stationary and non-stationary series. However, these techniques do not have built in inference or interpretation facilities. This limitation is addressed by fuzzy logic based systems, which are 'universal approximators' of non-linear functions [3]. Fuzzy logic time series modelling methods are broadly classified into those using *complex rule generation mechanisms* and *ad hoc data-driven models* [11] for automatic rule generation. The former employ hybrid methods, including neuro-fuzzy [6], [13], and probabilistic fuzzy [12] methods, while the latter utilize data covering criteria in example sets [15], [14]. Ad hoc data-driven models have the advantages of simplicity, speed and high performance and often serve as preliminary models that are subsequently refined using other methods [11], [2]. Moreover, since ad hoc models employ automatic rule generation, fuzzy rule bases can be built and updated with minimal effort. This enables the system to learn new patterns and potentially widen its application to complex domains like meteorology and astronomy.

Many existing fuzzy logic systems focus on the analysis of raw or return time series. We argue that, while this approach may produce a reasonable analysis and prediction, it is not optimal for non-stationary time series. Since such series are inherently noisy, the accuracy of analysis carried out on unprocessed data is impaired by its random components. There is, therefore, a need to provide a powerful analysis tool that provides predictions in terms of key dynamics of volatile time series. One such preprocessing tool is the wavelet transform, which has been shown to be useful in identifying the deterministic dynamics of financial processes [20]. Wavelets are mathematical functions that ‘cut’ up data into different frequency components, and study each component with a resolution matched to its scale [10], [9], [8]. Wavelet analysis decomposes a time series into several sub-series, which may be associated with particular time scales. The interpretation of features in complex financial time series is therefore made easy by first applying the wavelet transform and subsequently interpreting each individual sub-series [22], [18], [5].

Wavelet analysis has been previously combined with neural networks to provide time series forecasts [1]. Ho *et al.* describe a fuzzy wavelet network (FWN), where a fuzzy model is used to improve the accuracy of the wavelet sigmoid function [7]. Fuzzy logic and wavelets have also been used to model multiscale processes [4], where data collected at different sampling rates are decomposed using wavelets to facilitate multivariate analysis. Motivated by the effective preprocessing capability of wavelets and the predictive power of ad hoc fuzzy logic models, we present a unified framework for analyzing non-stationary time series. We investigate the effectiveness of wavelet-based preprocessing for a Takagi-Sugeno fuzzy logic system for forecasting non-stationary financial time series. Our framework provides fuzzy logic time series predictions based on components obtained from multiscale decomposition of a time series. We report results on intraday tick data for the British Pound (£) – US Dollar ($) exchange rate series and compare our results with a purely fuzzy-based prediction framework. The results indicate that a combined fuzzy-wavelet approach improves overall forecast performance.

3. Fuzzy-Wavelet Prediction Method

Given a time series X = {x_t, t = 1,…n}, where x_t is a value at discrete time *t*, we wish to predict the value at time t +1 (x_{t+1}). Our architecture comprises four main stages (Figure 1). The first stage involves data transformation where an equivalent volatility index, represented by the *return value*, $r_t = \log(x_t/x_{t-1})$, is calculated. In the second stage, the data is preprocessed: multiscale wavelet analysis is used to decompose the volatility series into various timescales, and for the elimination of random components. This is followed in the third stage by automatic rule generation for the wavelet timescale components to build a fuzzy logic model. This model is used to provide single step forecasts for each wavelet component. Fourth, individual predictions of wavelet component series are recombined to get an aggregate forecast.

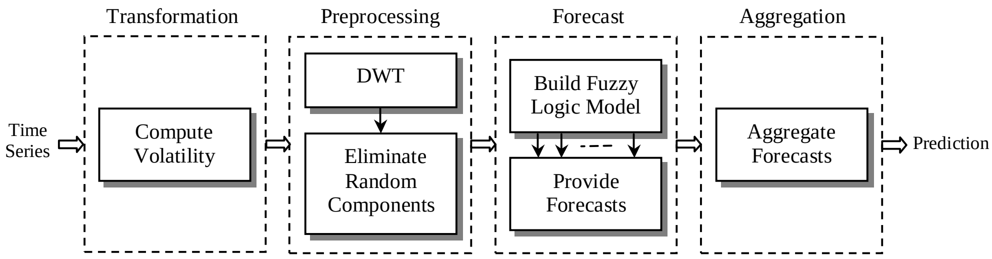


Figure 1. Architecture of fuzzy-wavelet prediction method.

3.1 Preprocessing using Wavelet Decomposition

Let *X* be a given time series. For a level *J* wavelet decomposition, *X* can be rewritten as:

$$X = \sum_{i=1}^{J} D_i + A_J \qquad (1),$$

where D_i are the details at different levels of decomposition and A_J is the wavelet approximation or smooth. The discrete wavelet transform (DWT) can be seen as a repeated *convolution* process. The discrete convolution process can be expressed by the following formula:

$$w * x_t = \sum_{i=-\infty}^{\infty} w_i x_{t-i} \qquad (2),$$

where x_t is the original signal and w is the low or high pass filter corresponding to the *prototype* wavelet. In practice, the DWT is implemented via a *pyramidal* algorithm [22] that, starting with the data x_t, filters it using high- and low-pass filters, keeps the output from the high-pass filter and repeats the above filtering operations on the output from the low-pass filter. In other words, a signal is split into an approximation (A) and a detail (D). The approximation (A) is then itself split into a second-level approximation and detail, and the process is repeated. Each successive recursion represents the highest to lowest frequency component of the original signal. Our system employs Daubechies six-coefficient filter banks to recursively convolve the original signal [9], [17].

The return series, r_t is preprocessed by generating additive level J wavelet decompositions, as in (1), based on the summarization algorithm described in [21]. The lower level wavelet decompositions are very high frequency components that generally correspond to noise in the input signal. We have experimented with removing these components, and obtaining a modified return series r_t' by using (3).

$$r_t' = r_t - \sum_{i=1}^{h} D_i \qquad (3),$$

where, r_t is the original return series and D_i are the wavelet components representing the noise in the series. The new series r_t' thus has a minimal influence of randomness.

3.2 Forecasting with a Fuzzy Rule-Based System

Given a time series, we derive a fuzzy rule base following a method due to Wang and Mendel [15], shown in Figure 2.

I. Determine the boundaries X^-, X^+ of example datasets i.e. the wavelet component under analysis and partition the datasets depending on the required rule granularity.

II. Pre-specify overlapping fuzzy sets covering expected data range for antecedents and singletons for consequents.

III. For each wavelet component, fuzzify data values by mapping them to pre-specified fuzzy sets. Given $x_t \in [X^-, X^+]$ for $\forall$t, generate membership functions for each data point:
  IF $x(t)$ (t = 1, 2, 3 …) THEN $0 \le \mu_x(x(t)) \le 1$
  IF $x(t) < X^-$ THEN $\mu_x(x(t)) = 0$
  IF $x(t) > X^+$ THEN $\mu_x(x(t)) = 1$

IV. Generate rules from successive sets of input and output data. For example, taking successive points *x(1), x(2), x(3), x(4)*, associated with fuzzy sets F1, F2… F4, respectively, we have:
  IF *x(1)* is F1 and *x(2)* is F2 and *x(3)* is F3 THEN *x(4)* is F4

V. Derive a degree, D, for each rule. The degree of a rule is the product of the membership functions of all its variables. For the above example,
  $D(R) = \mu_x(x(1))\, \mu_x(x(2))\, \mu_x(x(3))\, \mu_x(x(4))$

VI. Identify all conflicting rules i.e. rules with similar antecedents but different consequents.

VII. For each set of conflicting rules, select the rule with the highest degree and discard other rules.

VIII. Generate a rule base of all selected rules in VII.

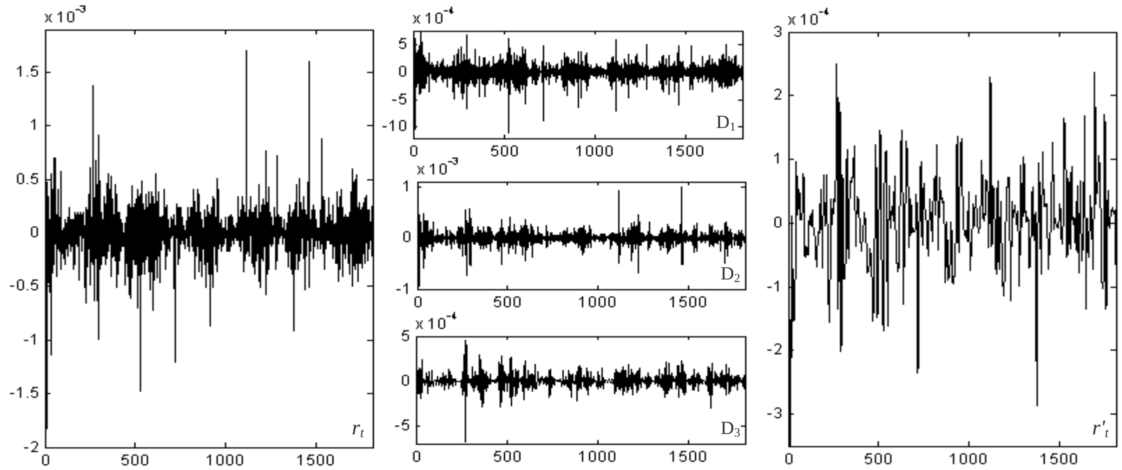
Figure 2. Fuzzy logic time series prediction algorithm.

An assumption has to be made about the consequents in a fuzzy rule base. Typically, in fuzzy control systems, a fuzzy set is replaced by either a constant or a linear relationship between inputs and outputs – this is referred to as the Takagi-Sugeno model [25]. The approximation of the consequent to a linear relationship makes the defuzzification stage redundant, saves computational time and throws light on the system behaviour.

4. Experimental Results

Dataset: The dataset used in our analysis comprises the £/$ exchange rate tick data, supplied by Reuters Financial News Service. All experiments are performed on five-minute compressed data. The training set consists of 1810 samples for the period ranging from March 10-18, 2004, and the test set is the £/$ exchange rate for March 19, 2004 (260 samples).

Analysis: The training data was transformed into its equivalent return series, r_t. A level-8 wavelet analysis (D_1-D_8 and A_8) was carried out, based on the length of the training dataset. Random components D_1, D_2 and D_3 were eliminated, resulting in a denoised return series, r_t' (Figure 3). A fuzzy logic model of higher order wavelet components (D_4-D_8 and A_8) was built and used to generate forecasts for each component. Finally, single step forecasts for individual wavelet components were combined to get a single prediction.



(a) Return series, r_t

(b) Outputs D_1, D_2, D_3 from the high pass filter

(c) Denoised return series, $r_t' = r_t - \sum_{i=1}^{3} D_i$

Figure 3. Preprocessing: wavelet denoising of the return series.

Evaluation: For evaluating our fuzzy-wavelet (F-W) framework, we present a comparison between predictions obtained for three series: (i) the raw time series, x_t, (ii) the return series, r_t, and (iii) the wavelet denoised return series, r_t'. The F-W prediction based on the wavelet-decomposed return series, r_t, provides a better prediction than that for the unprocessed time series, x_t, with a 7% reduction in the MSE (Table 1). However, the best fit to the data is obtained using the F-W prediction framework, where the return series r_t was denoised. This result indicates that random components constitute over 30% of the mean square error (MSE). We also compared the single step prediction performance of our model with the performance of a random walk model, assuming a Gaussian distribution. All our predictions (x_t, r_t, and r_t') outperformed the random walk model (Table 1). This suggests that even highly non-stationary time series exhibit certain regularities or patterns, which can be effectively modelled to give reasonable predictions.

Table 1. Summary of experimental results on £/$ exchange rate data.

Analysis Method	**MSE** ($\times 10^{-7}$)
Fuzzy model with x_t series (raw)	9.16
F-W model with r_t series (no denoising)	8.52
F-W model with r_t' series (denoised)	4.54
Random walk model	12.7

5. Conclusions

In this paper, we presented a time series analysis approach based on the combination of wavelet analysis and fuzzy modelling. We have shown how the DWT can be used to preprocess a non-stationary time series, thereby enabling the application of fuzzy logic forecasting techniques on non-random wavelet components. Initial experiments performed on high volatility intraday exchange rate data indicate that the combined fuzzy-wavelet approach performs better than pure fuzzy modelling. The multiscaling property of the DWT, it appears, enhances the prediction of high frequency, non-stationary and volatile financial time series. Future studies will involve the use of more sophisticated prediction models such as hybrid neuro-fuzzy methods. We also intend to explore techniques to enhance rule conflict resolution and to optimize the size of the rule base. Successful application of these improvements could result in better forecasts.

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